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AN INVESTIGATION OF AIRCRAFT HEATERS

XVI - DETERMINATION OF THE VISCOSITY OF EXHAUST

GASES FROM A GASOLINE ENGINE

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XVI - DETERMINATION OF THE VISCOSITY OF EXHAUST

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SUMMARY

The absolute viscosity of exhaust gases from a gaso-
line engine was measured at temperatures from 75° to
890° F and at compositions corresponding to fuel-air
ratios ranging from 0.0625 to 0.167. The viscosity was
found to be nearly independent of the fuel-air ratio and
within 6 percent of the value for dry air at the same
temperature. During the calibration tests measurements
of the viscosities of air, N_2, O_2, and CO_2 were also
obtained.

INTRODUCTION

Measurements of the viscosity of high-temperature
exhaust gases from an internal-combustion engine were
undertaken as part of a program to establish values of
certain physical properties of these gases. These prop-
erties - viscosity, thermal conductivity, heat capacity,
and density - are necessary in order to evaluate the heat-
transfer moduli used in the predictions of the performance
of exhaust gas and air heat exchangers.

The heat capacity and density of gaseous mixtures
can be closely approximated from calculations based on
measurements of these properties of the pure components.
There exists no experimental evidence, however, to prove
that any of the equations proposed for the calculation of
viscosity or thermal conductivity yield the correct re-
sults for an exhaust gas mixture at high temperatures.

Measurement of the viscosity of exhaust gases was first
undertaken. With the viscosity known, the thermal con-
ductivity can be approximately computed by means of a
relation derived from the kinetic theory of gases (refer-
ence 1).

The necessity of determining the viscosity at condi-
tions of elevated temperatures eliminates the use of many
of the precise methods already devised for use at room
temperature. The presence of water vapor in the gases
further reduces the number of available methods because
of the difficulty of retaining this component in making
volumetric flow measurements. The method of measuring
the time of viscous flow of a gas through a capillary
placed between two reservoirs, one at a continuously de-
creasing pressure and the other at a lower and constant
pressure, was selected as being most likely to meet all
the requirements.

A preliminary viscosimeter was constructed in order
to test the accuracy of the method. Tests at room tem-
perature on air, O_2, and CO_2 indicated that the method
was satisfactorily accurate. A second piece of apparatus
was then made to obtain measurements of viscosity at ele-
vated temperatures. The apparatus constant K was deter-
mined by calibration tests on air, O_2, and N_2, and vis-
cosity determinations were made on exhaust gases for sev-
eral fuel-air ratios at temperatures up to 890° F.

The present work is not considered complete, but the
results indicate that the method is satisfactory and that
the preliminary data may be useful.

SYMBOLS

P_0 atmospheric static pressure (lb)/(sq ft)

P_1 static pressure, in excess of P_0, of gas in
            viscosimeter at time ϵ_1 (lb)/(sq ft)

P_2 static pressure, in excess of P_0, of gas in
            viscosimeter at time θ_2 (lb)/(sq ft)

P_f measured static pressure, in excess of P_0, of gas in viscosimeter at conclusion of run $(lb)/(sq\ ft)$

l length of capillary tube (ft)

r inside radius of capillary (ft)

μ absolute viscosity of gas $(lb)(sec)/(sq\ ft)$

V_1 volume of tubing between zero level of manometer and stopcock 2 (cu ft)

V_2 viscosimeter-reservoir volume (cu ft)

V_3 volume of tubing between capillary and outlet stopcock 3 (cu ft)

θ_1 time at which gas pressure in viscosimeter is P_1 (sec)

θ_2 time at which gas pressure in viscosimeter is P_2 (sec)

$K = \dfrac{\pi\ r^4}{8\ l V_2}$ calibration constant for viscosimeter

Re Reynolds number $= \dfrac{u d \rho}{\mu}$

Re_{max} Reynolds number for flow through capillary

    (at time θ_1) $= \dfrac{u_{max}\ d\rho}{\mu}$

ρ mass density of gas $(lb)(sec^2)/(ft^4)$

d inside diameter of capillary tubing (ft)

u_{max} maximum cross-sectional mean velocity of gas between θ_1 and θ_2 $(ft)/(sec)$

u mean cross-sectional velocity of gas $(ft)/(sec)$

B constant in Sutherland equation,

$$\mu = \frac{B T^{1/2}}{1 + \dfrac{C}{T}} \left(\frac{lb\ sec}{ft^2\ {}^{\circ}R^{1/2}} \right)$$

C constant in Sutherland equation, $\mu = \dfrac{BT^{1/2}}{1 + \dfrac{C}{T}}$ ($^{\circ}R$)

ANALYSIS

In the case of the experiments reported herein, the fluid flows from a reservoir in which the pressure is continuously decreasing through the capillary tube into a region of constant pressure (atmospheric). The pressure in the upstream reservoir decreases only because of the flow out through the capillary.

Based on the Navier-Stokes differential equation of motion for viscous flow and neglecting acceleration factors, which are negligible for the condition of small pressure gradient along the capillary tube, the expression for the variation of pressure in the reservoir with time as derived in reference 2 is:

$$\frac{1}{2P_0} \log_e \left[\left(\frac{P_2 + 2P_0}{P_1 + 2P_0} \right) \frac{P_1}{P_2} \right] = \frac{\pi r^4}{16 \mu l V_2} (\theta_2 - \theta_1) \qquad (1)$$

Solving equation (1) for the gas viscosity μ yields:

$$\mu = -\left(\frac{\pi r^4}{8 l V_2} \right) \frac{P_0 (\theta_2 - \theta_1)}{\log_e \left[\left(\dfrac{P_2 + 2P_0}{P_1 + 2P_0} \right) \dfrac{P_1}{P_2} \right]} \qquad (2)$$

where

$$\frac{\pi r^4}{8 l V_2} = K \qquad (3)$$

Solving equation (2) for K yields:

$$K = \frac{\mu}{P_0 (\theta_2 - \theta_1)} \log_e \left[\left(\frac{P_2 + 2P_0}{P_1 + 2P_0} \right) \frac{P_1}{P_2} \right] \qquad (4)$$

The term K is a constant for the particular piece of apparatus used for the determination of the viscosity of the gas. The magnitude of K may be established in two ways: (1) By directly measuring r, l, and V_2 and computing K from equation (3); and (2) by calibrating

the apparatus with tests on a gas of known viscosity and computing K from equation (4).

APPARATUS AND METHODS

Description of apparatus.- The final viscosimeter (fig. 1) was constructed entirely of Pyrex to fit it for medium-temperature measurement. The volume of the cylindrical reservoir was 50.1 cubic inches; the capillary was 35 inches long and 0.0183 inch in diameter before coiling. The capillary was wound into a 2-inch diameter helix so that the entire viscosimeter could be placed inside an insulated cylindrical hot-air furnace. Because it was impracticable to measure directly the gas temperatures in the reservoir and capillary, these temperatures were approximated by means of three (chromel-alumel) thermocouples which were inserted into wells in the reservoir, two more thermocouples in the hot-air stream around the capillary, and one thermocouple in each of the ducts that carried the hot air in and out of the furnace. Two small-bore tubes with stopcocks were added to provide a separate evacuation line and inlet for the gas samples. Both the volume V_1 of the tubing between the manometer and the reservoir, and the volume V_3 between the capillary and outlet stopcock were held to minimum values by the use of small tubing in order to reduce the corrections to the final measured pressure; the corrections depend on the magnitudes of these volumes and on the corresponding temperatures of the gas in these volumes. (See the appendix.)

The values of capillary radius r and length l, and reservoir volume V_2 necessary for determination of the viscosimeter constant K by the direct method involving equation (3) were obtained by measuring the length before coiling and by calculating the reservoir volume from the weight of water and the capillary radius from the weight of mercury contained. The value of r obtained in this manner for a tube with bore irregularities is not exactly the value which should be used in the capillary-flow equation since the equation involves r^4 and the method of measuring r^2. The value of K obtained by this method, however, was used only as a check on the value obtained by the indirect method (equation (4)) which consisted of calibration of the apparatus with tests on a gas of known viscosity; namely, air,

The viscosimeter was fastened to a wooden stand for convenient handling and thoroughly cleaned with alcoholic potassium hydroxide, hot nitric acid, and distilled water. The reservoir was packed in an insulating material (ground asbestos) and the entire apparatus was set up in a constant-temperature room. The air used in the calibration tests was dried and purified before it entered the reservoir. The samples of O_2 and N_2 used in other runs were introduced directly without purification.

Preliminary runs.- The first two series of runs, one on air and one on CO_2, were made at room temperature, at initial pressures of 2 to 7.5 inches of mercury (measured above atmospheric pressure with a mercury manometer), and with times of efflux of 105 to 210 seconds. The results showed a decrease in the apparent value of K with an increase in pressure. The value approached a constant, however, at the low pressures and concomitant low efflux rates.

In order to obtain more accurate measurements at low pressures, the mercury manometer was replaced with one of similar design but with a longer column and using Ellison* fluid. A series of runs was made at room temperature on air at a range of pressures equivalent to 3 to 25 inches of water. These runs showed that there was no apparent deviation in K over this initial pressure range. Tests on CO_2 were not carried out because of the slight solubility of CO_2 in Ellison fluid.

Medium-temperature runs on exhaust gases.- A series of runs on exhaust gases then was made at a temperature range of 78° to 890° F, at a range of fuel-air ratios from 0.0602 to 0.158, and at pressures equivalent to 3 to 28 inches of water. The composition of the exhaust gases at various fuel-air ratios was determined by Orsat analyses.

Calibration runs at room temperature.- A series of calibration runs then was carried out to determine if the medium-temperature runs had affected the apparatus constant K. These runs were made on air, O_2, and N_2 at room temperature, at a pressure range equivalent to 3 to 28 inches of water, and at times of efflux from 60 to 315 seconds.

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*A mineral oil plus red dye, sp. gr. = 0.835 at 75° F.

Check runs at medium temperatures.- A series of runs
also was made to check the value of K at medium tempera-
tures. The runs were made on air, O_2, and N_2 at a tem-
perature range of 75° to 868° F and with pressure condi-
tions and times of efflux similar to those of the calibra-
tion runs.

RESULTS AND DISCUSSION

Preliminary runs.- The results of the preliminary
runs with the mercury manometer indicated variation in
the apparent value of K with change in pressure. This
variation may be attributed to Reynolds number effect for
flow through curved passages. When K is plotted as a
function of Re_{max} (fig. 2) the results of the runs on
CO_2 coincide with the results of the two series of tests
on air. This relation between K and Re_{max} shown in
figure 2 agrees qualitatively with the work of C. M. White
(reference 3) on steady flow of fluids through curved
pipes. From a study of the data on the flow of three dif-
ferent fluids (air, oil, and water) White concluded that
the static pressure drop along curved pipes could be cal-
culated from the laminar-flow equation for straight pipes

if the value of the dimensionless ratio $Re\left(\dfrac{d}{D}\right)^{1/2}$ were

less than 11.6 where Re is Reynolds number, d is the
inside diameter of the pipe, and D is the diameter of
the coil. From this relation the maximum value of Reynolds
number for which K should be constant was about 120,
although for the case of flow under decreasing pressure it
might be expected that the value of Re_{max} at time θ_1
could be somewhat higher than 120 before the effect on K
was apparent. The experimental data show that K does
decrease at values of Re_{max} only slightly above 120.

Calibration runs at room temperature.- The variations
in pressure and times of efflux in these tests (see table
1) resulted in no discernible effect on the values of K
obtained. The average deviations of the individual runs
from the mean of all runs of the three series was 0.35
percent for 85 runs on air, 0.26 percent for 20 runs on
N_2, and 1.0 percent for 19 runs on O_2. The mean value
of K for each of the three series was the same: namely,

$K = 14.7 \times 10^{-13}$; while the value obtained by direct meas-
urements was 15.3×10^{-13}. The value of K used in the
viscosity calculations for other gases was that obtained
by indirect measurement — that is, 14.7×10^{-13}.

Check runs at medium temperatures.- For purposes of
comparison, the value of K determined from low-pressure
(difference) measurements on air at room temperature was
used in conjunction with the pressure-time measurements
of the medium temperature runs to calculate the viscosi-
ties of N_2, O_2, and air at higher temperatures. (These
values are compared in fig. 3 with curves drawn through
the points obtained by previous investigators.) This
value of K was used because it is based on the viscos-
ity of air at room temperature, a subject that has been
very carefully investigated, and because room-temperature
measurements are subject to less error than higher-
temperature measurements. The curves of the values ob-
tained for the viscosities of O_2 and N_2 at room tem-
perature coincide with the curves obtained by previous
investigators, but at higher temperatures, the curves for
O_2 and N_2 as well as for air, lie slightly above the
curves of previous investigators. The maximum deviation
of the experimental curves from the curves obtained by
previous investigators are approximately 1 percent for
air and O_2 and 2.5 percent for N_2. (See table 2 for
summary of data.)

Exhaust gas measurements.- The results of the exhaust
gas measurements are given in table 3 and figure 4. These
results indicate that up to 890° F the viscosity of ex-
haust gases from gasoline engines is less than 6 percent
below that of air at the same temperature and is only
slightly affected by changes in fuel-air ratio. The pur-
pose of the following discussion is to indicate: (1) Why
the viscosity of exhaust gas mixtures should be approxi-
mately the same as the viscosity of air at the tempera-
tures of these measurements and closer at higher tempera-
tures, and (2) why changes in the fuel-air ratio produce
only slight changes in the viscosity of the exhaust-gas
mixtures.

Over the range 75° to 1600° F the viscosity curve
for oxygen lies about 10 percent above that for air;
while the curves for all the gases that replace oxygen
in the combustion mixture lie below the air curve. (See
fig. 5.) Although the exact viscosity of the mixture of

gases cannot be accurately determined from a knowledge
only of the composition of the mixture and the viscosities
of its constituents, the data clearly indicate that the
viscosity of exhaust gases should be close to and probably
below that of air at the same temperature. Furthermore,
the percentage deviation in the magnitudes of the viscos-
ities of the principal combustion products, CO_2 and
water vapor, from air at the same temperature decreases
considerably with temperature while the viscosities of
O_2, N_2, CO, and H_2 retain their relative positions with
respect to air. This relation indicates that, at higher
temperatures, the values of the viscosity of air and the
exhaust-gas mixtures should tend to converge slightly.

Further evidence that at high temperatures the vis-
cosities of the exhaust gas mixtures are only slightly
below the viscosity of air at the same temperature is ob-
tained by use of the Sutherland equation

$$\mu = \frac{BT^{1/2}}{1 + \dfrac{C}{T}} \qquad (5)$$

where B and C are constants for each gas or mixture
and T is the absolute temperature. This equation, which
has been experimentally verified over a wide range of
temperatures for pure gases by several investigators,
also appears to hold well for mixtures (reference 4).
Rearrangement of the equation to

$$T = B \left(\frac{T^{3/2}}{\mu} \right) - C \qquad (6)$$

gives a straight line, if the equation is valid, when T
is plotted against $\dfrac{T^{3/2}}{\mu}$. Plots of this type for air and
the exhaust gas mixtures (fig. 6) give straight lines
which, when extrapolated to 1500° F, indicate that the
viscosity of the lean mixtures is only 2.0 percent below,
and the rich mixtures 3.6 percent below, the viscosity of
air at 1500° F.

The slight effect of the fuel-air ratio on the vis-
cosity of the exhaust gases is made apparent by a con-
sideration of the effect of the fuel-air ratio on the

composition of the exhaust gases, as shown in the following table:

Fuel-air ratio	Percentage composition by volume			
	CO_2	CO	O_2	H_2O (calculated)
0.167-0.143	4.8	12	0.7	
.100- .091	6.7	7.7	1.4	
.063- .059	11.0	.1	2.8	12

If the fuel-air ratio is decreased beyond the optimum, the CO_2 content is increased at the expense of the CO. This change tends to reduce slightly the viscosity of the mixture, but the reduction is opposed by the increase in the O_2 content. If the fuel-air ratio is increased beyond the optimum, practically all the O_2 is removed from the air, and the steam and H_2 contents are slightly increased. This change might tend to reduce the viscosity, but it is opposed by the decrease in the CO_2-CO ratio. In general, although these changes in composition do not produce opposing effects of equal magnitude, all the changes in composition are small and their net effect on the viscosity are so negligible that the viscosity of the exhaust gases is practically equal to the viscosity of air at the same temperature.

CONCLUSIONS

1. Measurements of the viscosity of exhaust gases from a gasoline engine at temperatures from 75° to 890° F give values that are 3 to 6 percent below the viscosity of air at the same temperature.

2. Extrapolation of the results by Sutherland's equation indicates that at the temperature at which the exhaust gases leave the engine, about 1500° F, the viscosity of exhaust gas mixtures is only 2 to 4 percent below that of air at the same temperature.

3. Variation of the engine fuel-air ratio from 0.063
to 0.167 had very little effect on the viscosity of the
exhaust gases.

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     Berkeley, Calif., April 1944.

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APPENDIX

ACCURACY AND CORRECTIONS

Accuracy of the Measurements of Viscosity

The accuracy of the viscosities obtained from the present measurements is determined by the following factors:

1. The accuracy of the viscosity value used to determine the apparatus constant K

2. The validity of the postulates made in the derivation of the flow equation (2)

3. The accuracy of measurement of the quantities P_0, P_1, P_2, θ_1, and θ_2

4. The magnitude of the temperature variation during the test runs

5. The temperature gradient along the capillary tube

An analysis of the magnitude of these estimated errors indicates that at room temperature the individual runs should be within ±0.3 percent of the mean of all the runs of the series and that the error in the mean value should be

less than ±0.3 percent. At higher temperatures, where tem-
perature gradients and fluctuations are larger, the maximum
deviations from the mean value and the error of the mean
value both may be of the order of ±2.5 percent, if there is
only a small number of runs in the series. The experimental
results (figs. 3 and 4) indicate that the deviations and the
errors are of approximately the magnitude estimated, except
that at medium temperatures the errors always appear to be
positive. Comparison with other investigations indicates
that the experimental values obtained herein are slightly
higher than previously reported. On this basis, the viscos-
ities of the exhaust gas mixtures are probably about 1 to 2
percent above the true value.

Corrections Applicable to Equation (2)

Equation (2) postulates that the static pressure drop
at any point between the reservoir and the atmosphere is due
only to the viscous drag at the wall of the capillary tube.
Actually other mechanisms cause additional energy losses and
contribute to the over-all pressure drop. Briefly, these
additional mechanisms are manifested as:

(a) Initial acceleration of the gas in the capillary at
time (θ_1) of opening of stopcock 3

(b) Inlet pressure drop which includes loss due to con-
traction and also pressure drop due to acceleration

(c) Pressure drop due to change in velocity distribu-
tion along entrance length

(d) Pressure drop due to acceleration caused by change
in density of fluid along the tube

(e) Pressure drop due to expansion at end of capillary
tube

An analysis of these pressure-drop components indicates
that their magnitudes are small (less than the experimental
error) and that equation (2) can be used without appreciable
error.

Correction to Measurement of Final Pressure

The measured final pressure P_f had to be corrected to
obtain the value of P_2 to be substituted into equation (2)

for calculation of the fluid viscosity. This correction is necessary because it is impossible to measure the gas pressure P_2 existing in the reservoir at time θ_2. At time θ_2, as the outlet stopcock was closed and the watch simultaneously stopped, the gas in the bulb continued to flow through the capillary until the pressure in the small volume V_3 was raised from P_0 to P_f. When stopcock 2 in the manometer line was opened in order to measure the reservoir pressure at the end of the run, part of the gas in V_1 which was still under pressure P_1, escaped into the reservoir. This flow continued until the pressure P_f was established throughout V_1, V_2, and V_3.

The correction to be applied to P_f is obtained by application of the perfect gas law to the pertinent volumes, pressures, and weights of gas. Because the pressure differences and volumes V_1 and V_3 were small, the maximum correction of P_f to P_2 was only of the order of 0.1 inch of water.

TABLE 1.- RESULTS OF CALIBRATION RUNS

Number of runs	Gas used	Temperature ($^\circ$F)	Viscosity, μ $\left(\dfrac{lb/sec}{sq\ ft}\right)$	Range[a] of P_1 (in. water)	Range of θ	K (b)	Mean deviation (percent)
85	Air	74	3.83×10^{-7}	3.0-28	60-215	14.7×10^{-13}	0.35
19	O_2	79	4.29	6.3-25	120-180	14.7	1.0
20	N_2	75	3.67	5.4-25	120-315	14.7	.26

[a]Ellison fluid was used in the manometer for the measurements.

[b]The value of K calculated from direct measurement[c] of r, l, and V_2 was $K = 15.3 \times 10^{-13}$. The value of $K = 14.7 \times 10^{-13}$ was taken as the viscosimeter constant.

[c]The measurement of r was made before the capillary was coiled.

TABLE 2.- RESULTS OF CHECK RUNS AT MEDIUM TEMPERATURES

Gas	Temperature[a] (°F)	Number of runs	Viscosity, μ $\left(\dfrac{lb/sec}{sq\ ft}\right)$ (b)	Mean deviation (percent)
Air	256	9	4.78×10^{-13}	1.5
	431	6	5.58	1.6
	468	9	5.78	1.0
	598	4	6.24	3.0
	621	5	6.43	1.4
	868	9	7.30	1.5
				1.5 average
O_2	401	8	6.12	1.1
	474	7	6.60	.6
	594	9	7.14	1.2
	761	3	7.84	.8
				1.0 average
N_2	401	10	5.28	0.8
	502	7	5.76	1.4
	598	12	6.03	1.6
	755	10	6.68	.8
				1.2 average

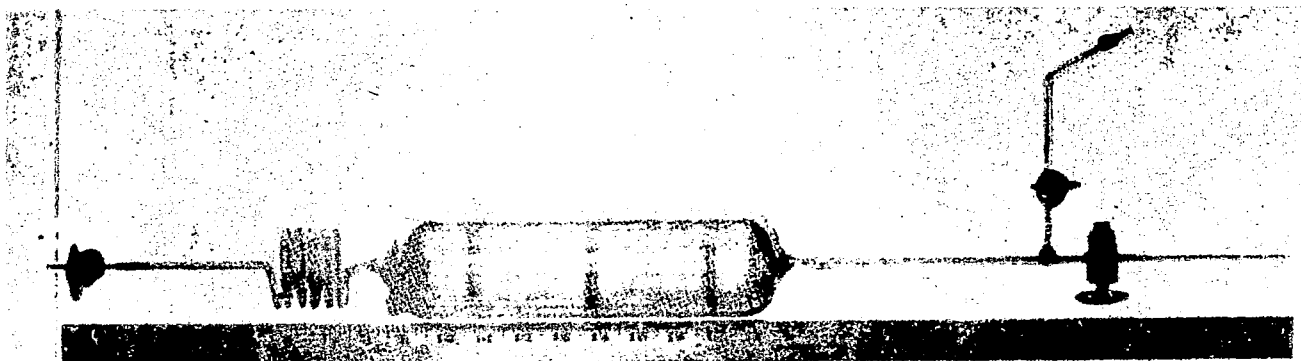[a]Temperature indicated by thermocouple number 4 in reservoir.

[b]These measured values were obtained by use of viscosimeter constant K based on air measurements at 76° F.
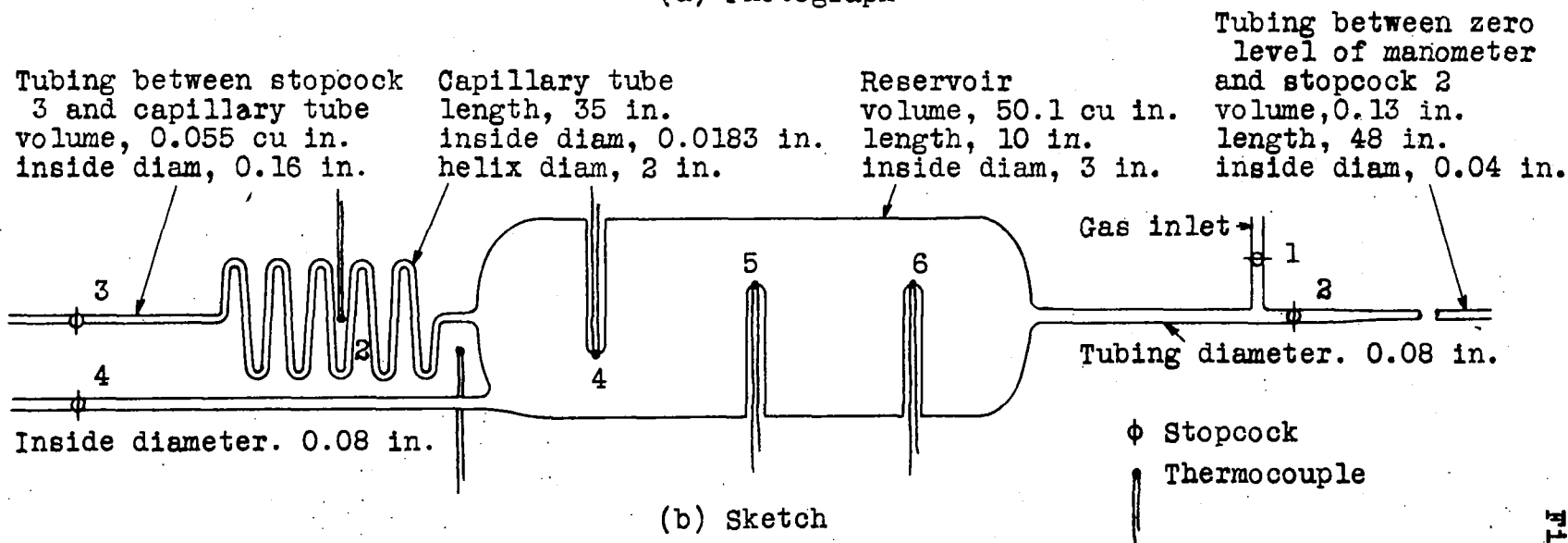
TABLE 3.- RESULTS OF MEASUREMENTS ON EXHAUST GASES

Temper- ature (°F)	Number of runs	Fuel-air ratio	Viscosity, μ $\left(\dfrac{lb/sec}{sq\ ft}\right)$	Mean deviation (percent)
[a]78	8	0.158	3.55×10^{-7}	0.8
[a]78	3	.075	3.65	.2
352	2	.152	5.01	.6
377	9	.100	5.10	1.0
378	6	.060	5.14	.8
488	4	.078	5.62	2.0
490	4	[b]R	5.47	1.3
490	2	.060	5.54	1.1
502	4	.060	5.66	2.0
590	11	.094	6.01	.6
598	4	.060	6.14	.9
599	9	.152	5.91	.7
672	4	.106	6.30	1.1
695	8	.060	6.47	1.0
800	10	[b]R	6.74	2.4
828	8	.106	6.83	1.4
890	9	[b]R	7.08	.9
				1.3 (average)

[a]The preliminary viscosimeter was used for the two room-temerature tests; all other measurements were made with the final viscosimeter.

[b]The three sets of measurements that have a fuel-air ratio denoted as R were very rich mixtures for which the fuel-air data were not recorded.

(a) Photograph

Tubing between stopcock
3 and capillary tube
volume, 0.055 cu in.
inside diam, 0.16 in.

Capillary tube
length, 35 in.
inside diam, 0.0183 in.
helix diam, 2 in.

Reservoir
volume, 50.1 cu in.
length, 10 in.
inside diam, 3 in.

Tubing between zero
level of manometer
and stopcock 2
volume, 0.13 in.
length, 48 in.
inside diam, 0.04 in.

Gas inlet

Tubing diameter. 0.08 in.

Inside diameter. 0.08 in.

φ Stopcock
Thermocouple

(b) Sketch

Figure 1.   Medium temperature viscosimeter.

Fig. 1
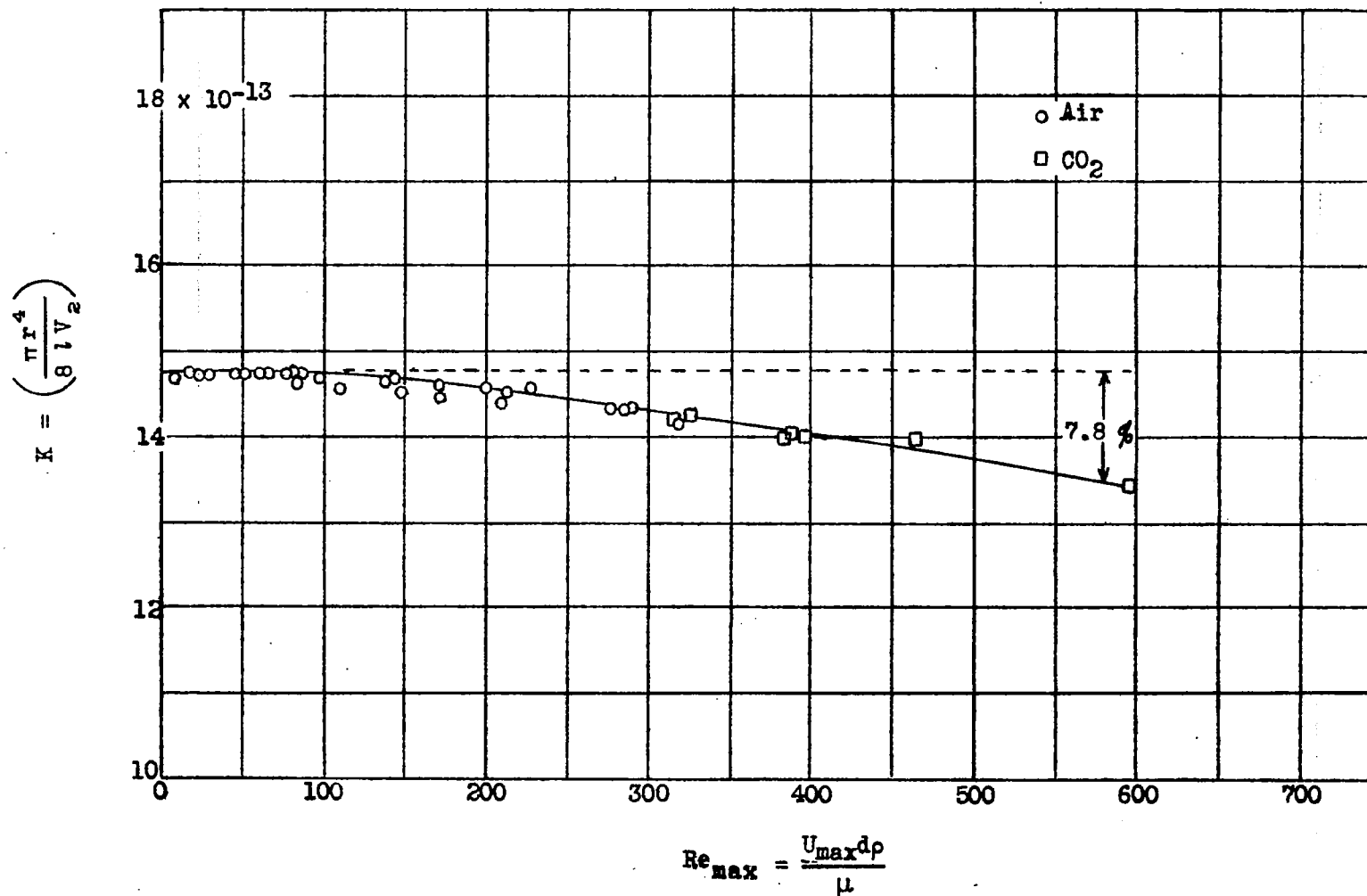
Figure 2.- Viscosimeter constant as a function of Reynolds number.

$$Re_{max} = \frac{U_{max}d\rho}{\mu}$$

$$K = \left(\frac{\pi r^4}{8 l V^2}\right)$$

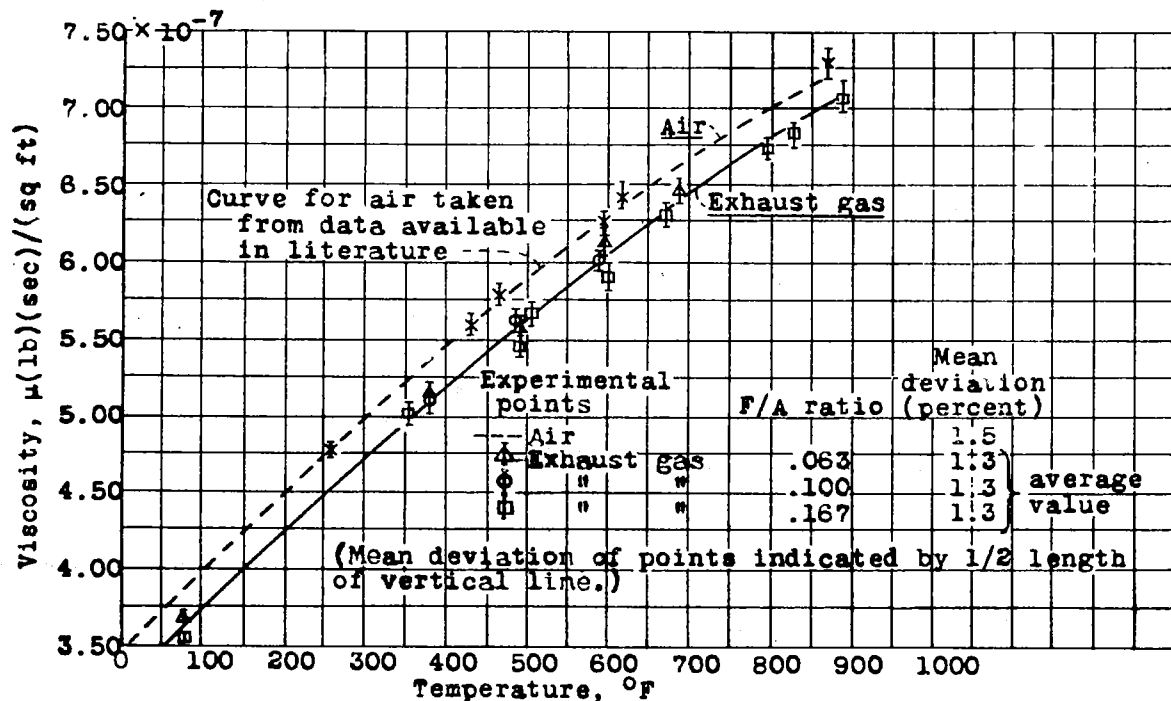Fig. 2
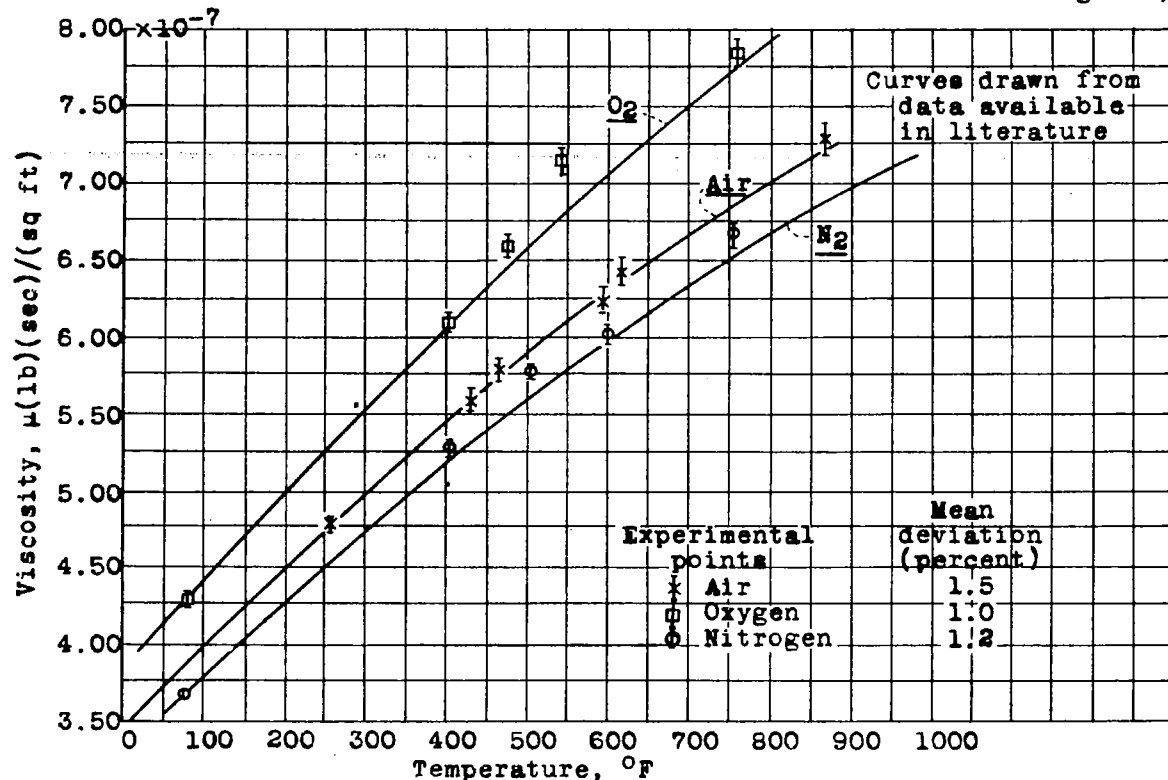
Figure 3.- Viscosity of air, oxygen, and nitrogen.

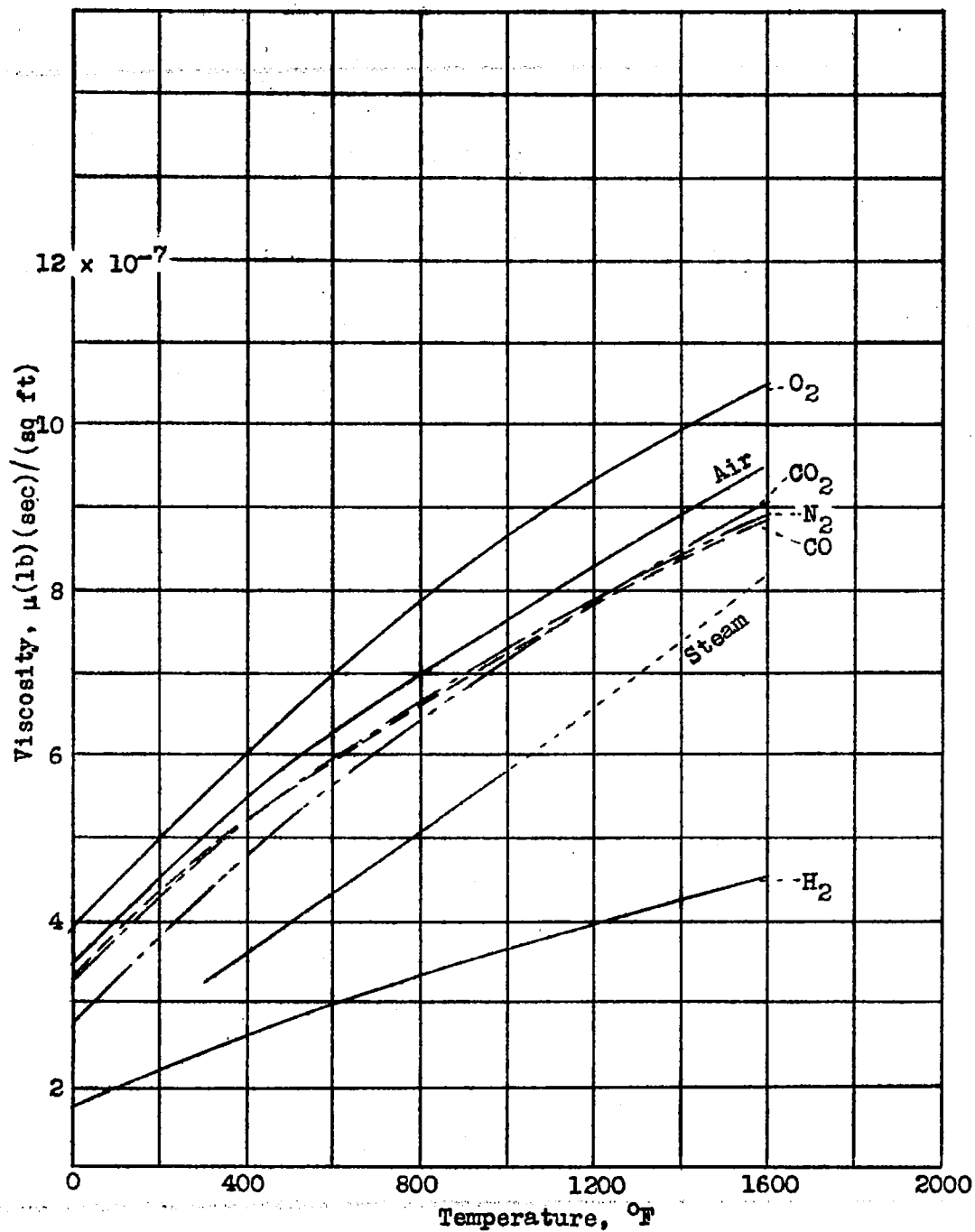Figure 4.- Viscosity of air and exhaust gas at various temperatures and fuel-air ratios.

Figure 5.- Viscosity of principal exhaust-gas constituents.
(Data from reference 1.)

$$B = 2.30 \times 10^{-8} \; \frac{lb \cdot sec}{ft^{2} \cdot {}^{o}R^{1/2}}$$

$$C = 260 \; {}^{o}R$$

$$B = 2.32 \times 10^{-8} \; \frac{lb \cdot sec}{ft^{2} \cdot {}^{o}R^{1/2}}$$

$$C = 200 \; {}^{o}R$$
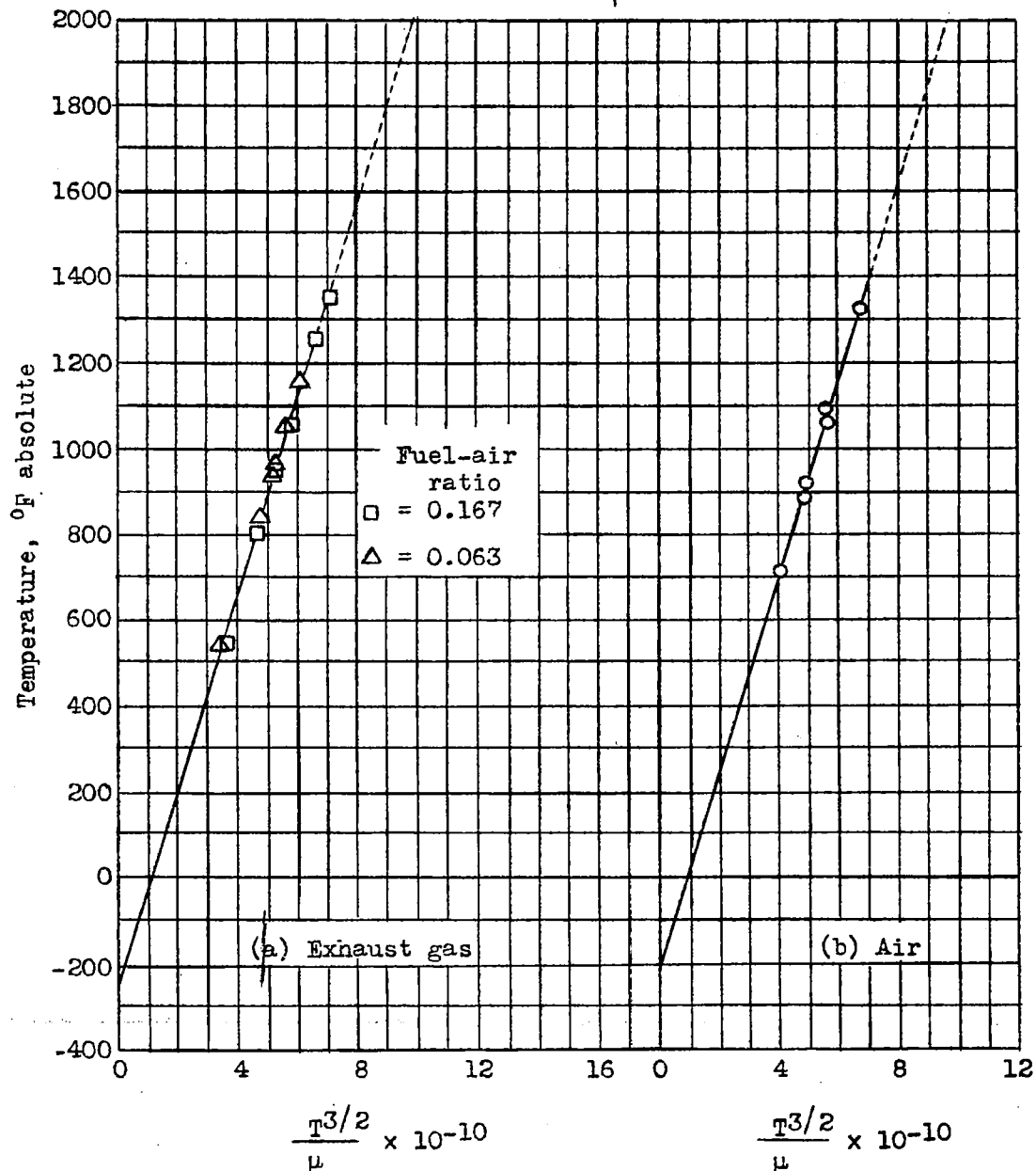
Figure 6.- Sutherland-equation plot, $T = \left(\dfrac{BT^{3/2}}{\mu}\right) - C$